Proofs and Natural Deduction Games

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- It turns out that we can use the inference rules in a game. This game helps us prove new things.
- (We will learn the game once we complete the next phase)

Natural Deduction Games

- Truth tables are helpful because they give us a straightforward test to determine whether an argument is valid.

- They also show us whether statements are logically equivalent.

- But how do we build new arguments? How do we take a gappy invalid argument, and fill in the gaps?

Game #1

Constructing Proofs

If the Astros make the playoffs, then the Braves will not win the pennant.
If the Cubs retain their manager, then the Braves will win the pennant.
The Astros will make the playoffs. Therefore, the Cubs will not retain their manager.

The Goal is to derive the conclusion from the premises alone, using the rules of inference. (Hint: it helps to look for the conclusion in the premises first. Think of it as though you are trying to "unlock" the conclusion you want)

First, put the argument into letters to get the structure.

1) If A, then not-B
2) If C, then B
3) A /not-C

The goal of the game is to build an argument with the last statement as the conclusion. We use our inference rules MP, MT, DS, and HS to build the path.

First, put the argument into letters to get the structure.

1) If A, then not-B
2) If C, then B
3) A /not-C

In this case, the goal is to derive not-C.
Can you find something that we can convert to not-C in the premises?
What inference rule would we use in order to convert it?

1) If A, then not-B
2) If C, then B
3) A /not-C

Now we make new steps based on the inferences available to us.

4. not-B (how did I get this one?) (1, 3, MP)

1) If A, then not-B
2) If C, then B
3) A /not-C

Now we make new steps based on the inferences available to us.

4. not-B (how did I get this one?) (1, 3, MP)

5. not-C (how did I get this one?) (2, 4, MT)

Thus we reach our goal: derive the conclusion, prove the argument valid.

Another example:

1. F or (if D, then T)
2. not-F
3. D /T

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1. F or (if D, then T)
2. not-F
3. D /T

Let's check our work:

1. F or (if D, then T)
2. not-F
3. D /T
4. if D, then T (how did we get this one?) (1, 2 DS)
5. T (how did we get this one?) (3,4, MP)

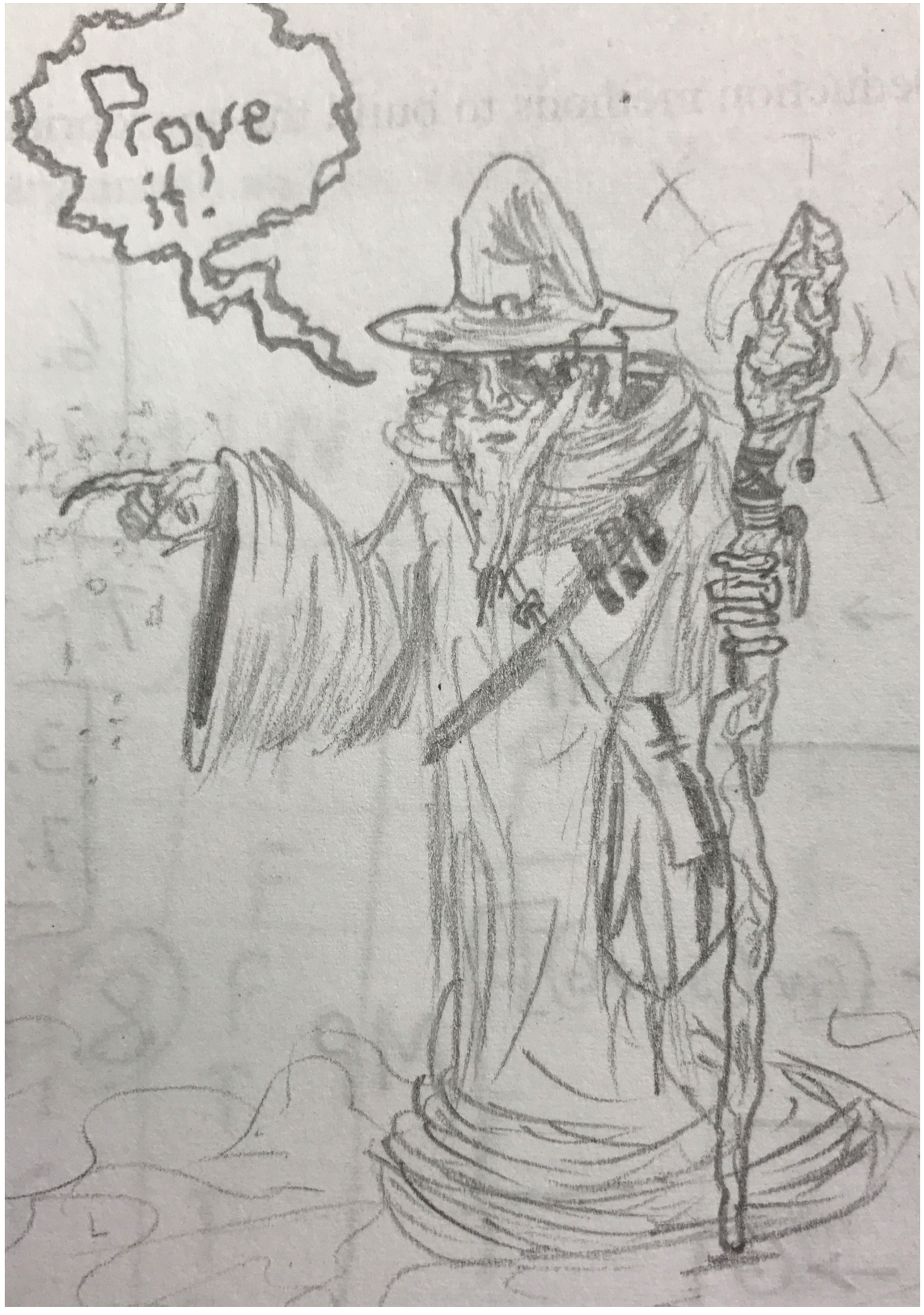
1. If A, then B
2. If not-A, then (C or D)
3. not-B
4. not-C /D

Let's check our work:

1. If A, then B
2. If not-A, then (C or D)
3. not-B
4. not-C /D
5. not-A (how did we get this one?) (1, 3 MT)
6. C or D (how did we get this one?) (2, 5 MP)
7. D (how did we get this one?) (4, 6 DS)

Some more examples: http://youtu.be/Gblt1aZUpLk (and textbook, chapter 7)

The evil logic wizard has issued a challenge. Can you solve each of the following puzzles?



III. Use the first four rules of inference to derive the conclusions of the following symbolized arguments.

★(1) 1. $\sim C \supset (A \supset C)$
2. $\sim C$ / $\sim A$

(2) 1. $F \vee (D \supset T)$
2. ~F
3. D / T

(3) 1. $(K \bullet B) \vee (L \supset E)$
2. $\sim(K \bullet B)$
3. $\sim E$ / $\sim L$

★(4) 1. $P \supset (G \supset T)$
2. $Q \supset (T \supset E)$
3. P
4. Q / $G \supset E$

(5) 1. $\sim W \supset [\sim W \supset (X \supset W)]$
2. $\sim W$ / $\sim X$

(6) 1. $J \supset (K \supset L)$
2. $L \vee J$
3. $\sim L$ / $\sim K$

★(7) 1. $\sim S \supset D$
2. $\sim S \vee (\sim D \supset K)$
3. $\sim D$ / K

(8) 1. $A \supset (E \supset \sim F)$
2. $H \vee (\sim F \supset M)$
3. A
4. $\sim H$ / $E \supset M$

(9) 1. $\sim G \supset (G \vee \sim A)$
2. $\sim A \supset (C \supset A)$
3. $\sim G$ / $\sim C$

★(10) 1. $N \supset (J \supset P)$
2. $(J \supset P) \supset (N \supset J)$
3. N / P

(11) 1. $G \supset [\sim O \supset (G \supset D)]$
2. $O \vee G$
3. $\sim O$ / D

(12) 1. $\sim M \vee (B \vee \sim T)$
2. $B \supset W$
3. $\sim\sim M$
4. $\sim W$ / $\sim T$

★(13) 1. $R \supset (G \vee \sim A)$
2. $(G \vee \sim A) \supset \sim S$
3. $G \supset S$
4. R / $\sim A$

(14) 1. $(L \equiv N) \supset C$
2. $(L \equiv N) \vee (P \supset \sim E)$
3. $\sim E \supset C$
4. $\sim C$ / $\sim P$

(15) 1. $\sim J \supset [\sim A \supset (D \supset A)]$
2. $J \vee \sim A$
3. $\sim J$ / $\sim D$

★(16) 1. $(B \supset \sim M) \supset (T \supset \sim S)$
2. $B \supset K$
3. $K \supset \sim M$
4. $\sim S \supset N$ / $T \supset N$

(17) 1. $H \vee (Q \vee F)$
2. $R \vee (Q \supset R)$
3. $R \vee \sim H$
4. $\sim R$ / F

(18) 1. $\sim A \supset (B \supset \sim C)$
2. $\sim D \supset (\sim C \supset A)$
3. $D \vee \sim A$
4. $\sim D$ / $\sim B$

★(19) 1. $\sim G \supset [G \vee (S \supset G)]$
2. $(S \vee L) \supset \sim G$
3. $S \vee L$ / L

(20) 1. $H \supset [\sim E \supset (C \supset \sim D)]$
2. $\sim D \supset E$
3. $E \vee H$
4. $\sim E$ / $\sim C$

(21) 1. $\sim B \supset [(A \supset K) \supset (B \vee \sim K)]$
2. $\sim J \supset K$
3. $A \supset \sim J$
4. $\sim B$ / $\sim A$

★(22) 1. $(C \supset M) \supset (N \supset P)$
2. $(C \supset N) \supset (N \supset M)$
3. $(C \supset P) \supset {\sim}M$
4. $C \supset N \quad / \; {\sim}C$

(23) 1. $(R \supset F) \supset [(R \supset {\sim}G) \supset (S \supset Q)]$
2. $(Q \supset F) \supset (R \supset Q)$
3. ${\sim}G \supset F$
4. $Q \supset {\sim}G \quad / \; S \supset F$

(24) 1. ${\sim}A \supset [A \vee (T \supset R)]$
2. ${\sim}R \supset [R \vee (A \supset R)]$
3. $(T \vee D) \supset {\sim}R$
4. $T \vee D \quad / \; D$

★(25) 1. ${\sim}N \supset [(B \supset D) \supset (N \vee {\sim}E)]$
2. $(B \supset E) \supset {\sim}N$
3. $B \supset D$
4. $D \supset E \quad / \; {\sim}D$